Van der Waerden/Schrijver-Valiant like Conjectures and Stable (aka Hyperbolic) Homogeneous Polynomials : One Theorem for all .

Leonid Gurvits*

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Abstract

Let $p(x_1,...,x_n) = p(X), X \in R^n$ be a homogeneous polynomial of degree n in n variables. Such polynomial p is called **H-Stable** if $p(z_1,...,z_n) \neq 0$ provided the real parts $Re(z_i) > 0, 1 \leq i \leq n$. This notion from *Control Theory* is closely related to the notion of *Hyperbolicity* intensively used in the *PDE* theory.

The main theorem, presented in this talk, states that if $p(x_1,...,x_n)$ is a homogeneous **H-Stable** polynomial of degree n, $deg_p(i)$ is the maximum degree of the variable x_i, $C_i = \min(deg_p(i), i)$ and $p(x_1, x_2, ..., x_n) \geq \prod_{1 \leq i \leq n} x_i; x_i > 0, 1 \leq i \leq n$ then the following inequality holds

$$\frac{\partial^n}{\partial x_1 ... \partial x_n} p(0,...,0) \geq \prod_{1 \leq i \leq n} \left(\frac{C_i - 1}{C_i}\right)^{C_i - 1} \geq \frac{n!}{n^n} \tag{1}$$

This inequality is a vast (and unifying) generalization of the van der Waerden conjecture on the permanents of doubly stochastic matrices as well as the Schrijver-Valiant conjecture on the number of perfect matchings in k-regular bipartite graphs. These two famous results correspond to the **H-Stable** polynomials which are products of linear forms.

The main goal of the talk is to give a reasonably complete proof of inequality (1). Our proof is relatively simple and "noncomputational"; it actually slightly improves Schrijver's lower bound, and uses very basic properties of complex numbers and AM/GM inequality.

Undergraduate and graduate students are encouraged to attend.

*Los Alamos National Laboratory Email: `gurvits@ lanl.gov`.